Screening in anyon gas

1997 J. Phys. A: Math. Gen. 30 L821

(http://iopscience.iop.org/0305-4470/30/24/002)

J. Phys. A: Math. Gen. **30** (1997) L821–L828. Printed in the UK PII: S0305-4470(97)86203-8

LETTER TO THE EDITOR

Screening in anyon gas

Subir Ghosh
Physics Department, Dinabandhu Andrews College, Calcutta 700084, India

Received 9 September 1997

Abstract. Anyon gas with interparticle (retarded) Coulomb interaction has been studied. The resulting system is shown to be a collection of dressed anyons, with a screening factor introduced in their spin. Close structural similarity with the Chern–Simons construction of anyons has helped considerably in computing the screening effect. Finally, the present model is compared with the conventional Chern–Simons construction.

1. Introduction

The possible existence of particles having arbitrary spin and statistics were proved quite some time ago [1, 2]. However, dynamical model building, on the other hand, has proved to be rather controversial, with the dispute still continuing. Basically there are two broad lines along which the models are conceived: (i) the Chern–Simons (CS) construction [3], where a point charge is coupled to CS electrodynamics. Removal of the auxiliary (or statistical) CS gauge field renders the particle anyonic. (ii) The construction of minimal anyon field equations, where one starts from very general physical postulates, such as the mass shell and Pauli–Lubanski condition for the particle [4]. A variant of the latter scheme is the spinning particle model [5–7], with which we are concerned in the present letter. The connection between the latter two is elaborated in [8].

It is important to point out that individually both models represent anyons. The controversy arises as regards to the nature of the CS gauge field in (i). The contention of [3], that the only effect of the CS gauge field is to influence the particle statistics and nothing else, has been debated strongly in [9]. Also the CS scheme fails in the relativistic theory, relevant for cosmic string problems.

Our result in this letter shows conclusively that *anyons in the presence of genuine interparticle Coulomb interaction, are dressed as far as their fractional spin (and hence the statistics parameter) is concerned*. The *screened* spin, $S = \alpha j$, with $\alpha = -\frac{Q^2}{16\pi^2\epsilon_0 mc^2}$, whereas in the spinning particle models [5, 6], the spin is $S = j$, where j is the Lagrangian spin parameter in (1). Here Q and m are the charge and mass of the anyon, c is the velocity of light and ϵ_0 is a characteristic property of the vacuum, (to be elaborated later). The dimensionless quantity α is the screening factor. We have considered a two-particle system but generalization to a many-particle system is straightforward.

Also the other interesting feature of the model is its structural similarity with CS construction [2]. We show that in the slow-moving and large-mass particle limit, the Coulomb field is structurally identical to the CS gauge field solution in [2], with the identification of the CS θ parameter, $\theta = -\frac{4\pi\epsilon_0 mc^2}{j}$. The crucial difference lies in the qualitative nature of the CS gauge field and the Coulomb field considered here. The former is sort of a fictitious gauge field [10], that couples to the fictitious charge of the particle, whereas the latter is the real Coulomb field, responsible for the Lorentz force between particles. Apart from this, there is the usual logarithmic Coulomb potential. Note that in the conventional CS scheme, the particles are endowed with the fractional spin $S = \frac{Q^2}{4\pi\theta_{\mathrm{CS}}}$ where θ_{CS} is the arbitrary CS parameter and Q is the fictional charge that couples to the CS statistical gauge field. One also recovers the logarithmic Coulomb interaction [10]. The above-mentioned identification helps us to use the CS results directly to compute explicitly the screening factor α. The connection between the conventional CS scheme and our model will be elaborated later. Although, some of the major results of this paper have appeared in [11], the implications and consequences, discussed in the conclusion, were not emphasized before.

The spinning particle Lagrangian proposed by us in [6] is $(c = 1)$,

$$L = \sqrt{m^2u^2 + \tfrac{1}{2}j^2\sigma^2 + mj\epsilon^{\mu\nu\lambda}u_\mu\sigma_{\nu\lambda}} \tag{1}$$

where the velocity and canonical momenta are defined as

$$u^\mu = \frac{\mathrm{d}r^\mu}{\mathrm{d}\tau} \qquad \sigma^{\mu\nu} = \Lambda^\mu_\lambda\dot{\Lambda}^{\lambda\nu} \tag{2}$$

$$P^\mu = -\frac{\partial L}{\partial u_\mu} \qquad S^{\mu\nu} = -\frac{\partial L}{\partial\sigma_{\mu\nu}}. \tag{3}$$

$(r^\mu, \Lambda^{\mu,\nu})$ is a Poincaré group element, as well dynamical variables with the property, $\Lambda\Lambda^T = \Lambda^T\Lambda = g$, where g is the Minkowski metric $g^{00} = -g^{11} = -g^{22} = 1$.

The action in (1), $\int L\,\mathrm{d}\tau$ is invariant under reparametrizations of the arbitrary parameter $\tau \to \tau' = f(\tau)$. The details of the constraint analysis can be found in [6]. We will use the relevant Dirac brackets (DB) as and when necessary. Let us briefly demonstrate the appearance of the arbitrary phase. The set of second-class constraints (SCC) and first-class constraints (FCC) are

$$S^{\mu\nu}P_\nu \approx 0 \qquad \Lambda^{0\mu} - \frac{P^\mu}{m} \approx 0 \tag{4}$$

$$P^2 - m^2 \approx 0 \qquad \epsilon^{\mu\nu\lambda}S_{\mu\nu}P_\lambda - mj \approx 0. \tag{5}$$

Let us transform the set of SCCs in (4) to strong equality. The induced DB relevant to us,

$$\{\Lambda^{\mu\nu}, S^{12}\} = (\Lambda^{\mu1}g^{\nu2} - \Lambda^{\mu2}g^{\nu1}) + \frac{1}{m^2}(P^\nu P^1\Lambda^{\mu2} - P^\nu P^2\Lambda^{\mu1} - P_\rho\Lambda^{\mu\rho}P^1g^{\nu2} + P_\rho\Lambda^{\mu\rho}P^2g^{\nu1}) \tag{6}$$

in the particle rest frame, $P^i = 0$, $P^0 = m$, reduces to

$$\begin{aligned}&\{\Lambda^{11}, S^{12}\} = \Lambda^{12} \qquad \{\Lambda^{22}, S^{12}\} = -\Lambda^{21}\\ &\{\Lambda^{12}, S^{12}\} = -\Lambda^{11} \qquad \{\Lambda^{21}, S^{12}\} = \Lambda^{22}.\end{aligned} \tag{7}$$

Using the rest frame Λ's, i.e. $\Lambda^{01} = \Lambda^{02} = 0$, $\Lambda^{00} = 1$ we obtain the relations,

$$\begin{aligned}&\Lambda^{10} = \Lambda^{20} = 0 \qquad (\Lambda^{12})^2 + (\Lambda^{11})^2 = (\Lambda^{21})^2 + (\Lambda^{22})^2 = 1\\ &\Lambda^{11}\Lambda^{21} + \Lambda^{12}\Lambda^{22} = 0.\end{aligned} \tag{8}$$

Hence, in the reduced phase space we can parametrize the remaining independent variables by,

$$\Lambda^{12} = \cos\phi \qquad \Lambda^{11} = \sin\phi \qquad S^{12} = \frac{\partial}{\partial\phi} \tag{9}$$

where S^{12} is the Pauli–Lubanski scalar in the rest frame,

$$\left.\frac{\epsilon^{\mu\nu\lambda}S_{\mu\nu}P_\lambda}{m}\right|_{\text{rest frame}} = \frac{S_{12}P_0}{m} = S_{12}.$$

Also from counting the number of independent degrees of freedom in phase space we see that out of three each (independent) $S^{\mu\nu}$ and $\Lambda^{\mu\nu}$ variables one each of S and Λ remain, since out of the set of six SCCs in (4) only *two* from each set (totalling four) are independent. So far the FCCs have remained intact. This is exactly the parametrization employed by Plyushchay in [5]. This Λ variable (or equivalently ϕ), gives rise to the arbitrary phase. This is consistent with the fact [12] that specifically in $2+1$ dimensions, the number of (phase space) degrees of freedom for a particle with fixed mass and spin is the same as that of a massive spinless particle. Here the remaining degrees of freedom, i.e. ϕ and S^{12}, can be removed by choosing a gauge for the Pauli–Lubanski FCC. However, the effect of the spin variables present in (1) manifest itself in the non-trivial DBs, which gives rise to the spin contribution in the total angular momentum.

Let us elaborate on the last point, which also brings about a qualitative change in the nature of the system, that is the P and T violation. Apart from (6), the other crucial modification occurs in the r_μ-DB,

$$\{r_\mu, r_\nu\} = \frac{-S_{\mu\nu}}{m^2}. \tag{10}$$

This non-trivial algebra necessitates a change in the conventional expression of the angular momentum,

$$J_\mu = \epsilon_{\mu\nu\lambda}(r^\nu P^\lambda + \tfrac{1}{2}S^{\nu\lambda}) = \epsilon_{\mu\nu\lambda}r^\nu P^\lambda + S_\mu. \tag{11}$$

This is the conserved angular momentum, which can be directly inferred from the Lagrangian (1) [6]. The extra term is needed to maintain the proper angular momentum algebra. This is the fractional spin term. On the other hand, as we are using DBs, the set of SCCs can now be incorporated as strong relations and we can totally discard the spin variables by the relation,

$$S_\mu = \frac{j}{\sqrt{P^2}}P_\mu = \frac{j}{m}P_\mu. \tag{12}$$

(One can check this relation to be true in the DB sense. A subtle point to note is that $P^2 = m^2$ is allowed only after the DBs have been computed.) But in the $r_\mu - P_\nu$ subspace of variables, the full algebra with the identification in (12), the DB (10) and the angular momentum (11) constitute a magnetic monopole [13]. Basically, the simultaneous presence of electric and magnetic charges is the origin of the P and T violation [14]. In fact, heuristically it is remarked [15] that this non-commuting r-algebra is an analogue of the non-commuting velocities in the presence of a magnetic monopole, with the monopole strength replaced by $\frac{j}{m^2}$. A point-charge magnetic monopole interaction term can also be introduced [16] in a (non-relativistic) Lagrangian, to simulate the same effect which is done here by the spin variables. The interaction term, being a total derivative, does not affect the equations of motion but changes the angular momentum spectrum in a similar way [16].

As is clearly shown in [13], different first-order Lagrangians are allowed, whose symplectic structures are identical to the one used here. Our Lagrangian is one such (possibly more down to earth) alternative, being a coordinate space second-order one. In fact, it is quite akin to the $(3+1)$-dimensional spinning particle model, proposed in [17]. As has been emphasized in [13], the vital elements for a description of anyons are the mass shell condition and the symplectic structure. All the models mentioned, as well as the present one, are equivalent in this respect.

The complexities in the quantum CS gauge theory of anyons provided the impetus for the search of more economical schemes and the spinning particle (or symplectic or magnetic monopole) approach is indeed a viable one.

Note an interesting departure in the constraint structure from a parent (3+1)-dimensional model [18], where the spin (FC) constraint appeared as a combination of the SCCs $S^{\mu\nu}P_\nu \equiv 0$, due to some non-trivial algebraic identities. The latter are absent in $2+1$ dimensions, as a result of which the spin constraint comes here independently, i.e. it is not obtainable from the other FCC and SCCs, in (4) and (5).

Let us now move to the main body of our work. Hereafter we will use rationalized MKS units and keep c and $\hbar$ as they come. This will be useful for the comparison of the final results and churning out numerical estimates. We use the planar Coulomb law as

$$\boldsymbol{F}_{\text{Coul}} = \frac{Q_1 Q_2}{2\pi\epsilon_0 r}\boldsymbol{n} \tag{13}$$

where $\boldsymbol{r} = r\boldsymbol{n}$ is the separation between the charges Q_1 and Q_2, and ϵ_0 is the 'permittivity' of the vacuum. $\boldsymbol{F}_{\text{Coul}}$ denotes the force between the particles. This Coulomb law is compatible with the Gauss law in a plane, $\nabla\cdot\boldsymbol{E} = \frac{\rho}{\epsilon_0}$, where $\boldsymbol{E}$ and ρ are the electric field and charge density respectively. We introduce μ_0 and ϵ_0 as the 'permeability' and 'permittivity' of the vacuum, to keep the relations same as their (3+1)-dimensional counterpart. We only use the relation $\epsilon_0\mu_0 = \frac{1}{c^2}$. Denoting by [O] the dimension of O, we note that

$$[\epsilon_0] = \frac{C^2}{M(L/T)^2} \qquad [\phi] = \frac{M(L/T)^2}{C} \qquad [A_i] = \frac{M(L/T)}{C}.$$

Here, M, L, T, C are mass, length, time units and Coulomb respectively. ϕ and A_i are the scalar and vector potentials. We have the standard relations,

$$\boldsymbol{E} = -\nabla\phi - \dot{\boldsymbol{A}} \qquad \boldsymbol{B} = \nabla X\boldsymbol{A}.$$

Here $\boldsymbol{B}$ is the magnetic field.

We briefly show the construction of the relativistic Darwin Lagrangian for a system of two interacting point charges in a plane. As the results have appeared in [11], we simply incorporate c, ϵ_0 and μ_0 in their respective places. The retarded logarithmic potentials are,

$$\phi = \frac{1}{2\pi\epsilon_0}\int \mathrm{d}^2 r\,\rho\left(\boldsymbol{r}, t - \frac{r}{c}\right)\ln\frac{r}{r_0} \tag{14}$$

$$\boldsymbol{A} = \frac{\mu_0}{2\pi}\int \mathrm{d}^2 r\,\rho\left(\boldsymbol{r}, t - \frac{r}{c}\right)\boldsymbol{v}\ln\frac{r}{r_0} \tag{15}$$

where r_0 denotes some length scale where the potential due to a point charge vanishes. Expanding in terms of v the particle velocity and keeping terms up to $\mathrm{O}(\frac{v^2}{c^2})$, with the charge density $\rho = Q\delta(\boldsymbol{r} - \boldsymbol{r}_{\text{particle}})$, we obtain

$$\phi = \frac{Q}{2\pi\epsilon_0}\left[\ln\frac{r}{r_0} - \frac{1}{c}\left(r\ln\frac{r}{r_o}\right)^{\cdot} + \frac{1}{2c^2}\left(r^2\ln\frac{r}{r_0}\right)^{\cdot\cdot}\right]$$

$$\boldsymbol{A} = \frac{\mu_0}{2\pi}Q\boldsymbol{v}\ln\frac{r}{r_0}.$$

Performing a gauge transformation,

$$\phi \to \phi' = \phi - \frac{\partial f}{\partial t} \qquad \boldsymbol{A} \to \boldsymbol{A}' = \boldsymbol{A} + \nabla f$$

such that,

$$\frac{\partial f}{\partial t} = \frac{Q}{2\pi\epsilon_0}\left[-\frac{1}{c}\left(r\ln\frac{r}{r_0}\right)^{\cdot} + \frac{1}{2c^2}\left(r^2\ln\frac{r}{r_0}\right)^{\cdot\cdot}\right]$$

$$\nabla f = \frac{Q}{2\pi\epsilon_0}\left(-\frac{\boldsymbol{n}}{c}\ln\frac{r}{r_0} - \frac{\boldsymbol{n}}{c}\right) + \frac{Q}{4\pi\epsilon_0 c^2}\left(2\boldsymbol{n}r\ln\frac{r}{r_0} + \boldsymbol{r}\right)^{\cdot}$$

the retarded potential ϕ is reduced to the standard Coulomb form,

$$\phi' = \frac{Q}{2\pi\epsilon_0}\ln\frac{r}{r_0} \tag{16}$$

$$\boldsymbol{A}' = -\frac{Q}{2\pi\epsilon_0}\left[\frac{\boldsymbol{n}}{c^2}\left(\boldsymbol{n}\cdot\boldsymbol{v} + c\left(1+\ln\frac{r}{r_0}\right)\right) + \frac{\boldsymbol{v}}{c^2}\right]. \tag{17}$$

The interaction is simply of the minimal current-gauge field form $J_\mu A^{'\mu}$, where $J_0 = \rho = Q\delta(\boldsymbol{r}-\boldsymbol{r}_p)$, $\boldsymbol{J} = Q\boldsymbol{v}\delta(\boldsymbol{r}-\boldsymbol{r}_p)$ and $A^{'\mu}$ is the above set, (16) and (17). Thus, to $O(\frac{v^2}{c^2})$, the Lagrangian, or the Hamiltonian obtained just below, incorporates the effect of Coulomb interaction between two charges, taking into account the relativistic corrections via the retarded time.

The two-particle Darwin Hamiltonian is,

$$H = \frac{p^2}{m} - \frac{p^4}{4m^3c^2} + \frac{Q^2}{2\pi\epsilon_0}\ln\frac{r}{r_0} + \frac{Q^2}{2\pi\epsilon_0 c^2}\left[\frac{\boldsymbol{r}\cdot\boldsymbol{p}}{mr}\left(\frac{\boldsymbol{r}\cdot\boldsymbol{p}}{mr} + c\left(1+\ln\frac{r}{r_0}\right)\right) - \frac{p^2}{2m^2}\right]. \tag{18}$$

Note that the correction terms in $\boldsymbol{A}'$ are qualitatively different from their (3+1)-dimensional counterpart [19]. This has also induced the difference in the Hamiltonian correction terms.

So far the effect of the particle spin has not been taken into account. Now we do this via a non-canonical transformation. We rewrite (1),

$$L = c^2\sqrt{\left(\frac{m^2u^2}{c^2} + \frac{j^2\sigma^2}{2c^4} + \frac{mj}{c^3}\epsilon^{\mu\nu\lambda}u_\mu\sigma_{\nu\lambda}\right)} \tag{19}$$

with the dimensions of the phase-space variables being,

$$[u] = \frac{L}{T} \qquad [\sigma] = [\Lambda\dot{\Lambda}] = T^{-1} \qquad [S_{\mu\nu}] = [j] = \frac{ML^2}{T} \qquad [P_\mu] = \frac{ML}{T}.$$

With $P^2 = m^2c^2$ and $S^2 = 2j^2$, the DBs relevant to us are [6],

$$\{r^\mu, r^\nu\} = -\frac{S^{\mu\nu}}{m^2c^2} = -\frac{j}{m^3c^3}\epsilon^{\mu\nu\lambda}P_\lambda \qquad \{r^\mu, P^\nu\} = g^{\mu\nu} \qquad \{P^\mu, P^\nu\} = 0. \tag{20}$$

Invoking the quantization prescription that $\frac{\mathrm{i}}{\hbar}\{\mathrm{DB}\} \rightarrow [\mathrm{commutator}]$, we arrive at the following commutators,

$$[r^\mu, r^\mu] = \frac{\mathrm{i}\hbar}{m^2c^2}S^{\mu\nu} = \frac{\mathrm{i}\hbar j}{m^3c^3}\epsilon^{\mu\nu\lambda}P_\lambda \qquad [r^\mu, P^\nu] = -\mathrm{i}\hbar g^{\mu\nu} \qquad [P^\mu, P^\nu] = 0. \tag{21}$$

One can 'solve' the algebra by introducing the non-canonical transformation [5, 11],

$$r^i = q^i + \frac{j}{m^2c^2}\epsilon^{ij}P_j \qquad P^i = p^i \tag{22}$$

where (q, p) constitute a canonical pair with the non-zero commutator $[q^i, p^j] = -\mathrm{i}\hbar g^{ij}$. The transformation simulates the spin property of the particle, as it has originated from the non-trivial $[r^i, r^j]$ commutator in (21), which was crucial in producing the spin part of the total angular momentum. We will come to this point again. Note that although we have a canonical position coordinate q, the consequence of this is that q does not transform as a position vector. However, this departure can be quite small for slowly moving heavy particles.

This modifies H to

$$
\begin{aligned}
H_{\mathrm{spin}} = H\left(P^i = p_i, r^i = q^i + \frac{j}{m^2c^2}\epsilon^{ij}p_j\right) \\
= \frac{p^2}{m} - \frac{p^4}{4m^3c^2} + \frac{Q^2}{2\pi\epsilon_0}\Bigg[\ln\frac{q}{r_0}(1+\alpha) + \alpha(1+\alpha) \\
+ \frac{j}{mcq}\beta\left(1 - 2\alpha^2 - \alpha\ln\frac{q}{r_0}\right) + \left(\frac{j}{mcq}\right)^2\alpha^2\beta^2 - \left(\frac{j}{mcq}\right)^2\alpha\beta^2\Bigg] \qquad (23)
\end{aligned}
$$

where the two dimensionless variables α and β are,

$$
\alpha = \frac{\boldsymbol{q}\cdot\boldsymbol{p}}{mcq} \approx \mathrm{O}\left(\frac{v}{c}\right) \qquad \beta = \frac{\epsilon^{ij}q_ip_j}{mcq} \approx \mathrm{O}\left(\frac{v}{c}\right).
$$

Let us define,

$$
\begin{aligned}
A^i = \frac{Qj}{4\pi\epsilon_0 mc^2q^2}\epsilon^{ij}q_j = \sigma\frac{\epsilon^{ij}q_j}{q^2} \\
a^i = \left(1 - \alpha\ln\frac{q}{r_0} - 2\alpha^2\right)A^i
\end{aligned} \qquad (24)
$$

and rewrite H_s as,

$$
\begin{aligned}
H_s = \frac{1}{m}(\boldsymbol{p} - Q\boldsymbol{a})^2 + \frac{Q^2}{2\pi\epsilon_0}\Bigg[\ln\frac{q}{r_0} + \alpha\left(1 + \ln\frac{q}{r_0}\right) + \alpha^2 - \left(\frac{j}{mcq}\right)^2\alpha\beta^2 \\
+ \left(\frac{j}{mcq}\right)^2\alpha^2\beta^2\Bigg] - \left(\frac{Q_j}{4\pi\epsilon_0 mc^2}\right)^2\frac{Q^2}{mq^2}\left(1 - \alpha\ln\frac{q}{r_0} - 2\alpha^2\right)^2. \qquad (25)
\end{aligned}
$$

The identification [11] of our system with that of a point charge interacting with CS gauge field is now obvious. The α-independent term in α^i is the explicit solution of the CS gauge field. Hence we can identify [4],

$$
\theta = -\frac{Q}{\sigma} = -\frac{4\pi\epsilon_0 mc^2}{j} \qquad (26)
$$

where θ is the CS parameter in the CS Lagrangian,

$$
L_{\mathrm{CS}} = \frac{c\theta}{2}\int \mathrm{d}^2r\,\epsilon^{\mu\nu\lambda}\partial_\mu A_\nu A_\lambda.
$$

Also the magnetic flux connected to the charged particle is Φ, where

$$
\Phi = -\frac{Qj}{2\epsilon_0 mc^2}. \qquad (27)
$$

Note the Φ is of the proper dimension of magnetic flux. This is one of our cherished results, where we have been able to obtain Φ in terms of the spinning particle parameters by simply borrowing the CS result.

Let us now elaborate on the previously advertised dressing induced by the Coulomb interaction. Since we already identified our system with point charge CS system, the results of the latter can be directly used. According to CS theory [20], the physical states can be shown to be carrying an angular momentum eigenvalue S, which is related to the CS parameter $\theta_{\rm CS}$ by $S = \frac{e^2}{4\pi\theta_{\rm CS}}$. Here e is the fictional charge of the particle that couples to the CS gauge field to generate the anyon. This is the well known fractional spin. In our case,

$$S = \frac{e^2}{4\pi\theta_{\rm CS}} = -\frac{Q^2 j}{16\pi^2\epsilon_0 mc^2}. \tag{28}$$

Note that if $S = s\hbar$,

$$s = \frac{e^2}{4\pi\hbar\theta_{\rm CS}} = \frac{e}{2\theta_{\rm CS}(h/e)} = \frac{\Phi}{4\pi\Phi_0}$$

where Φ_0 is the flux quantum and Φ is obtained from (27).

However, in the case of the minimal spinning particle model [5, 6], due to the non-trivial $[r^i, r^j]$ commutator, the angular momentum is modified by the spin contribution in the following way,

$$J^\mu = -\epsilon^{\mu\nu\lambda} r_\nu p_\lambda - \frac{j}{\sqrt{p^2}} p^\mu. \tag{29}$$

Construction of the Pauli–Lubanski scalar $\boldsymbol{p}\cdot\boldsymbol{J} = -jmc$ clearly shows that the particle spin is just j. Hence in comparison with (28), we notice the extra parameters or dressings that have appeared as a result of the Coulomb interaction. This is the main result of the present work.

Let us now put the CS and our work in their proper perspectives. Our model of interacting anyons can be cast in the following form,

$$\mathcal{L} = \left[\sum_{i=1}^{2}\left(\frac{1}{2}mv_i^2 + e(\boldsymbol{v_i}\cdot\mathcal{A} - \mathcal{A}_0)\right) + \frac{c\theta_{\rm CS}}{2}\int {\rm d}^2 r\, \epsilon^{\mu\nu\lambda}\mathcal{A}_\mu\partial_\nu\mathcal{A}_\lambda\right] + \left[\sum_{i=1}^{2} Q(\boldsymbol{v_i}\cdot\boldsymbol{A} - \boldsymbol{A}_0)\right] \tag{30}$$

where for simplicity we have used non-relativistic expressions. The fictitious charge e and gauge fields $\mathcal{A}_\mu$ make the particles anyonic, and Q and A_μ are their genuine charges and Coulomb interaction. For consistency, when computing A_μ, one should consider the anyon spin as well. This will be complicated if retardation effects are to be taken into account.

On the other hand, we have started with a spinless interacting system with genuine charges and evaluated the (Darwin) Lagrangian with retardation effects duly taken care of. Subsequently we turn the whole system anyonic (via(22)), and obtain the screening effect. The fact that our interacting anyon system, in the lowest order, is structurally similar to the CS system, has made life easier, by allowing us to borrow previous results.

We now conclude with the following comments.

(i) We have considered a system of interacting anyons, following our spinning particle model and have shown that there is a screening effect in the anyon spin, arising from mutual Coulomb interactions.

(ii) We have shown how our system should be compared with the CS construction.

(iii) We have not used the $c = \hbar = 1$ convention and this has made some of the relations look clumsy. We have persisted with this since all the dimensions of electromagnetic quantities have been overhauled, as we have taken the planar logarithmic Coulomb potential to be fundamental.

(iv) Unless there is a proper definition of planar ϵ_0 with a numerical value, it is of no use to speculate about numerical estimates.

(v) Finally, it would be interesting to see if the CS construction described above reproduces this screening.

References

[1] Leinaas J M and Myrheim J 1971 *Nuovo Cimento* B **37** 1
[2] Wilczek F 1982 *Phys. Rev. Lett.* **49** 957
[3] See for example Wilczek F 1990 *Fractional Statistics and Anyon Superconductivity* (Singapore: World Scientific)
Arovas D P, Schrieffer R, Wilczek F and Zee A 1985 *Nucl. Phys.* B **251** 117
Hagen C 1985 *Phys. Rev.* D **31** 2135
Semenoff G W 1988 *Phys. Rev. Lett.* **61** 517
[4] Jackiw R and Nair V P 1991 *Phys. Rev.* D **43** 1933
Plyushchay M S 1991 *Phys. Lett.* B **262** 71
[5] Plyushchay M S 1991 *Int. J. Mod. Phys.* A **7** 7045
Chou C, Nair V P and Polychronakos P 1993 *Phys. Lett.* B **304** 105
[6] Ghosh S 1994 *Phys. Lett.* B **338** 235
Ghosh S 1995 *Phys. Rev.* D **51** 5827
[7] Gorbunov I V, Kuzenko S M and Lyakhovich S L 1997 *Phys. Rev.* D **56** 3744
[8] Ghosh S and Mukhopadhyay S 1995 *Phys. Rev.* D **51** 6843
[9] Jackiw R and Pi S-Y 1990 *Phys. Rev.* D **42** 3500
[10] Chen Y-H, Wilczek F, Witten E and Halperin B I 1989 *Int. J. Mod. Phys.* B **3** 1001
[11] Banerjee N and Ghosh S 1995 *Phys. Rev.* D **52** 6130
[12] Cortez J L and Plyushchay M S 1996 *Int. J. Mod. Phys.* A **11** 3331
[13] Jackiw R and Nair V P 1994 *Phys. Rev. Lett.* **73** 2007
[14] See for example Jackson J D 1975 *Classical Electrodynamics* (New York: Wiley)
[15] de Sousa Gerbert Ph 1990 *Nucl. Phys.* B **346** 440
[16] Forte S 1992 *Rev. Mod. Phys.* **64** 193
[17] Balachandran A P, Marmo G, Skagerstam B-S and Stern A 1983 *Gauge Symmetries and Fibre Bundles* (Springer)
[18] Hanson A J and Regge T 1974 *Ann. Phys., NY* **87** 498
[19] Landau L D 1975 *Classical Theory of Fields* (New York: Pergamon)
[20] See for example Jackiw R 1990 Topics in planar physics *Nucl. Phys.* B **18A** 107